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# PROBABILISTIC MODELING OF COMMON CHANNEL SIGNALING

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This report details preliminary models for a common-channel signaling system that sets up and tears down voice calls in a circuit-switched network. The initial Sections 1-3 present alternative detailed models for a single link between circuit-switched nodes. Section 4 outlines a heuristic procedure for calculating delays in a network; it makes use of an M/G/1 queueing approximation partially justified earlier. The simple illustrative problem addressed suggests approaches to a realistic network.

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PROBABILISTIC MODELING OF COMMON CHANNEL SIGNALING

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**Abstract and Summary**

This report details preliminary models for a common-channel signaling system that sets up and tears down voice calls in a circuit-switched network. The initial Sections 1-3 present alternative detailed models for a single signaling link between circuit-switched nodes. Section 4 outlines a heuristic procedure for calculating delays in a signaling network; it makes use of an M/G/1 queueing approximation partially justified earlier. The simple illustrative problem addressed in Section 4 suggests approaches to a realistic network.

## 1. INTRODUCTION

Common channel signaling is an out-of-band signaling method in which several data and/or voice channels use a common data channel to transmit signaling information; [cf. Skoog [1990]]. The demand for the signaling channel is generated by activity on the data and/or voice channels. For voice channels, examples of signals are call-setup messages and call-tear-down messages.

In this paper we introduce several different probability models for common channel signaling on the link between two connected nodes (switches). All the models represent the common signaling channel as a single server; cf. CCITT Study Group XI [1984]. A customer arriving at the server requires several types of services, somewhat spaced out in time. There is an initial service, such as a voice call setup, which must be completed before the customer can use the voice/data network. After completion of this initial service, however, the customer does not require further service from the common channel server (CCS) for a period perhaps approximately equal to the call length, but, after such a random time the customer *will* require another type of service from the CCS. Both service requests impose load on the CCS, thus increasing delays.

**Assumptions**

In this preliminary discussion we will assume that each arriving customer requires two types of service: an initial call setup service which requires a time $X$; and, after a period of time, a call tear-down service which requires a time $Y$. These services are assumed to be independent with fixed distributions. It is interesting to inquire as to whether the distribution of $X$ at least should depend upon the current network state, i.e. the difficulty of call setup.

In Section 2, a model is studied for the work at one common channel signaling server. The model allows the distributions of X and Y to be arbitrary. Equations for the moment generating function of the amount of work are given. A simple approximation is proposed for the long run average amount of work at the server. Simulation is used to study the effect of simplifying assumptions in the model and the approximation. In Section 3, two simple queueing network models for one common channel signaling server are described; the models have a product form limiting distribution. In Section 4 a simple example is given to illustrate how the approximation of Section 2 for one common channel server may be used to study a common channel signaling network. Appendix B briefly examines the possibility of using a specific voice/data network model, the CSNDAM, to generate loads for a common channel signaling network model.

## 2. A MODEL FOR THE AMOUNT OF WORK AT THE COMMON CHANNEL SERVER

There are two service centers. One service center is a single server queue representing the common channel server. The service discipline is first-come-first-served. The second service center consists of $K$ servers and represents calls in progress.

Customers arrive according to a Poisson process with rate $\lambda$. The $n^{th}$ call to arrive requires two service times from the CCS, $X_n$ and $Y_n$. Assume $\{X_n\}$ and $\{Y_n\}$ are independent sequences of independent identically distributed random variables. The distribution of $X_n$ will, in general, be different from that for $Y_n$. The $n^{th}$ call also requires a service time $C_n$ from the second service center. Assume $\{C_n\}$ are independent identically distributed exponential with mean $1/\upsilon$.

To simplify the state space of the initial model, we make the following assumptions: if a customer arrives when all $K$ servers in the second service center are busy, the customer is lost. If a customer arrives when there is at least one free server at service center 2, then it immediately queues at the CCS for an $X$-service. It also *simultaneously* starts a $C$-service at service center 2. This assumption is made for convenience: a $C$-service is actually a call, which cannot begin until the setup is complete, i.e. when the call completes its X-service. However, dutiful modeling at this level of detail complicates the state space (at least one must keep track of the numbers of each type of job in the queue and make an assumption concerning the type—setup or tear down—receiving service). In our model, after a customer finishes a $C$-service, it queues up at the CCS for a $Y$-service. When its $Y$-service is completed the call is finished. In this initial model it is conceptually and actually possible for a call to be completed (finish its $C$-service) before its setup ($X$-service) has been finished. The effect is minor when $E[X] \ll 1/\upsilon$, as is the case in practice. Adjustment to the model can be made to compensate for this effect but as previously noted dimension of the model state space is expanded.

Let $W_t$ be the total amount of work at the CCS at time $t$. Let $N_t$ be the number of busy servers at service center 2. Note that $\{N_t; t \geq 0\}$ is the number of customers in a M/M/K/K queue and its long-run marginal distribution is given by

$$\pi(n) = \lim_{t\to\infty} P\{N_t = n\} = \pi(0)\left(\frac{\lambda}{\upsilon}\right)^n \frac{1}{n!} \qquad n = 0,1,\dots,K \tag{2.1}$$

with $\pi(0) = \left[\sum_{n=0}^{K}\left(\frac{\lambda}{\upsilon}\right)^n \frac{1}{n!}\right]^{-1}$ ; that is, it is truncated Poisson.

### *a. A System of Equations for the Laplace Transform of the Amount of Work at the CCS*

Let

$$Q_n(\theta;t) = E\left[e^{-\theta W_t}; N_t = n\right].$$

Forward differential equations can be written for $\{Q_k(\theta;t)\}$. For example,

$$\begin{aligned} Q_0(\theta;t+h) &= E\left[e^{-\theta W_{t+h}}; N_{t+h} = 0\right] \\ &= \left\{E\left[\exp\{-\theta[W_t - h]\}; N_t = 0, W_t > h\right] + P\{W_t < h, N_t = 0\}\right\}[1 - \lambda h] \\ &\quad + E\left[\exp\{-\theta(Y + W_t - h)\}; N_t = 1\right]\upsilon h + o(h) \\ &= A(t;h)[1 - \lambda h] + \hat{Y}(\theta) Q_1(\theta;t)\upsilon h + o(h) \end{aligned} \tag{2.2}$$

where

$$A(t;h) = e^{\theta h} E\left[e^{-\theta W_t}; N_t = 0, W_t > h\right] + P\{W_t < h; N_t = 0\}$$

and $\hat{Y}(\theta) = E\left[e^{-\theta Y}\right]$ is the Laplace transform of a $Y$-service time. Let $F_0(x;t) = P\{W_t \le x; N_t = 0\}$.

$$\begin{aligned} A(t;h) &= e^{\theta h}\int_h^\infty e^{-\theta x} F_0(dx;t) + F_0(h;t) \\ &= e^{\theta h} Q_0(\theta;t) - e^{\theta h}\int_0^h e^{-\theta x} F_0(dx;t) + \int_0^h F_0(dx;t) \\ &= e^{\theta h} Q_0(\theta;t) - e^{\theta h}\left[p_0(0;t) + \int_{0^+}^h e^{-\theta x} F_0(dx;t)\right] + p_0(0;t) + \int_{0^+}^h F_0(dx;t) \end{aligned}$$

$$= e^{\theta h}Q_0(\theta;t) - p_0(0;t)\left[1 - e^{\theta h}\right] + \int_{0^+}^{h}\left(1 - e^{\theta(h-x)}\right)F_0(dx;t)$$

$$= (1 + \theta h)Q_0(\theta;h) - \theta h p_0(0;t) + o(h) \tag{2.3}$$

where $p_n(0;t) = P\{W_t = 0, N_t = n\}$.

Thus, rewriting (2.2)

$$Q_0(\theta;t+h) = [1 - \lambda h]\{(1+\theta h)Q_0(\theta;t) - \theta h p_0(0;t)\}$$
$$+ \upsilon h\hat{Y}(\theta)Q_1(\theta;t) + o(h). \tag{2.4}$$

Simplifying

$$Q_0(\theta;t+h) = [1 - (\lambda - \theta)h]Q_0(\theta;t) - \theta h p_0(0;t) + \upsilon h\hat{Y}(\theta)Q_1(\theta;t) + o(h). \tag{2.5}$$

Subtracting $Q_0(\theta;t)$ from both sides of the last equation, dividing by $h$ and letting $h \to 0$ results in the equation

$$\frac{\partial}{\partial t}Q_0(\theta,t) = -(\lambda - \theta)Q_0(\theta;t) + \upsilon\hat{Y}(\theta)Q_1(\theta;t) - \theta p_0(0;t). \tag{2.6}$$

Similar arguments result in the following system of equations.

For $1 \le n < K$

$$\frac{\partial}{\partial t}Q_n(\theta;t) = -(\lambda + n\upsilon - \theta)Q_n(\theta;t) + (n+1)\upsilon\hat{Y}(\theta)Q_{n+1}(\theta;t) \tag{2.7}$$
$$+ \lambda\hat{X}(\theta)Q_{n-1}(\theta;t) - \theta p_n(0;t);$$

$$\frac{\partial}{\partial t}Q_K(\theta;t) = -(K\upsilon - \theta)Q_K(\theta;t) + \lambda\hat{X}(\theta)Q_{K-1}(\theta;t) - \theta p_K(0;t) \tag{2.8}$$

where

$$p_n(0;t) = P\{W_t = 0, N_t = n\}$$

as before and

$$\hat{X}(\theta) = E\left[e^{-\theta X}\right],$$

the Laplace transform of an $X$-service time.

Assume $Q_n(\theta) = \lim_{t \to \infty} Q_n(\theta;t)$ exists. The system of equations satisfied by $\{Q_n(\theta)\}$ are

$$\theta p_0(0) = -(\lambda - \theta)Q_0(\theta) + \upsilon\hat{Y}(\theta)Q_1(\theta); \tag{2.9}$$

for $0 < n < K$,

$$\theta p_n(0) = \lambda\hat{X}(\theta)Q_{n-1}(\theta) - (\lambda + n\upsilon - \theta)Q_n(\theta) + (n+1)\upsilon\hat{Y}(\theta)Q_{n+1}(\theta); \tag{2.10}$$

and $$\theta p_K(0) = \lambda\hat{X}(\theta)Q_{K-1}(\theta) - (K\upsilon - \theta)Q_K(\theta) \tag{2.11}$$

where $p_n(0) = \lim_{t \to \infty} P\{W_t = 0, N_t = n\}$.

Let $\underline{p}(0)$ be a column vector whose $n^{th}$ entry is $p_n(0)$. Let $\underline{Q}(\theta)$ be a column vector whose $n^{th}$ entry is $Q_n(\theta)$. let $D(\theta)$ be a square matrix with $(K+1)$ rows with nonzero entries possible only on the lower diagonal, diagonal, and upper diagonal and with $i,j$ entries $D(i, j; \theta)$ as follows:

$$D(k,k-1;\theta) = \lambda\hat{X}(\theta) \qquad 1 < k \le K+1 \tag{2.12}$$
$$D(k,k;\theta) = \theta - \lambda - (k-1)\upsilon \qquad 1 < k \le K \tag{2.13}$$
$$D(1,1;\theta) = \theta - \lambda \tag{2.14}$$
$$D(K+1,K+1;\theta) = \theta - K\upsilon \tag{2.15}$$
$$D(k,k+1;\theta) = k\upsilon\hat{Y}(\theta) \qquad 1 \le k \le K. \tag{2.16}$$

The system of equations (2.9)-(2.11) can be rewritten as

$$\theta\underline{p}(0) = D(\theta)\underline{Q}(\theta). \tag{2.17}$$

For an example of $D(\theta)$ for $K = 2$, see Appendix A.

Let $\overline{N}_n(\theta)$ be the matrix $D(\theta)$ with the $n^{th}$ column replaced by the vector $\theta\underline{p}(0)$. From Cramer's rule,

$$Q_n(\theta) = \frac{\det(\overline{N}_n(\theta))}{\det(D(\theta))}. \tag{2.18}$$

Since the columns of $D(0)$ sum to zero, det $(D(0)) = 0$. Hence, a Taylor expansion of $D(\theta)$ about $\theta = 0$ yields

$$\det D(\theta) = \sum_{k=1}^{\infty} \frac{\theta^k}{k!} \frac{d^k}{d\theta^k} \det D(0) \equiv \theta d(\theta). \tag{2.19}$$

Further det $\overline{N}_n(\theta) = \theta b_n(\theta)$ where $b_n(\theta)$ is the determinant of the matrix $B_n(\theta)$ whose entries are the same as $D(\theta)$ except that its $n^{th}$ column is $\underline{p}(0)$. Thus

$$Q_n(\theta) = \frac{b_n(\theta)}{d(\theta)}. \tag{2.20}$$

### *b. A Stability Condition*

A stability condition for the queue can be obtained from the equation

$$1 = \sum_{n=0}^{K} Q_n(0) = \frac{1}{d(0)} \sum_{n=1}^{K+1} b_n(0). \tag{2.21}$$

In Appendix A it will be shown that

$$\sum_{n=1}^{K+1} b_n(0) = (-1)^K \left[\sum_{k=0}^{K} p_k(0)\right] \sum_{i=0}^{K} i!\binom{K}{i} \lambda^{K-i} \upsilon^i \tag{2.22}$$

and

$$d(0) = (-1)^{K-1} \left\{ \lambda E[X] \left( \left[ \sum_{i=1}^{K} \binom{K}{i} i! \lambda^{K-i} \upsilon^i \right] \right) - \left[ \sum_{i=0}^{K} \binom{K}{i} i! \lambda^{K-i} \upsilon^i \right] \right.$$

$$\left. + \upsilon E[Y] \left[ \sum_{i=0}^{K-1} (K-i) \binom{K}{i} i! \lambda^{K-i} \upsilon^i \right] \right\}. \tag{2.23}$$

Hence,

$$\left[\sum_{k=0}^{K} p_k(0)\right] = \left[\sum_{i=0}^{K} i!\binom{K}{i}\lambda^{K-i}\upsilon^i\right]^{-1} [-d(0)]. \qquad (2.24)$$

Since $0 < \sum_{k=0}^{K} p_k(0) < 1$, it follows that

$$0 < \sum_{i=0}^{K}\binom{K}{i}i!\lambda^{K-i}\upsilon^i - \left[\lambda E[X]\sum_{i=1}^{K}\binom{K}{i}i!\lambda^{K-i}\upsilon^i + \upsilon E[Y]\sum_{i=0}^{K-1}(K-i)\binom{K}{i}i!\lambda^{K-i}\upsilon^i\right] \quad (2.25)$$

Rewriting (2.25), a stability condition is

$$\sum_{i=0}^{K}\binom{K}{i}i!\lambda^{K-i}\upsilon^i > \lambda E[X]\sum_{i=1}^{K}\binom{K}{i}i!\lambda^{K-i}\upsilon^i + \upsilon E[Y]\sum_{i=0}^{K-1}(K-i)\binom{K}{i}i!\lambda^{K-i}\upsilon^i. \quad (2.26)$$

Note that when the stability condition is satisfied, $d(0) > 0$ for $K$ even and $d(0) < 0$ for $K$ odd.

c. *A Numerical Procedure to Evaluate Long-run Average Work at the Common Channel Server*

The Laplace transform of the limiting amount of work at the CCS is

$$E\left[e^{-\theta W}\right] = \sum_{n=0}^{K} Q_n(\theta) = \frac{1}{d(\theta)}\sum_{n=1}^{K+1} b_n(\theta) \equiv \frac{b(\theta)}{d(\theta)}. \qquad (2.27)$$

We conjecture that if the stability condition (2.26) is satisfied, then $d(\theta)$ has $K$ positive distinct roots, $0 < \theta_1 < \theta_2 < \ldots < \theta_K$. At each of these roots the equation

$$0 = b(\theta_i) \qquad i = 1,\ldots,K \qquad (2.28)$$

is an equation involving $\{p_n(0); n = 0, 1, \ldots, K\}$. These equations in addition to the equation

$$1 = \frac{b(0)}{d(0)} \qquad (2.29)$$

yield $(K+1)$ independent equations from which the $\{p_n(0)\}$ can be obtained.

To obtain an expression for $E[W]$ differentiate (2.27) with respect to $\theta$ and evaluate at $\theta = 0$ which results in

$$E[W] = -\left[\frac{b'(0)d(0) - b(0)d'(0)}{d(0)^2}\right]. \tag{2.30}$$

The derivatives of the determinants of the matrices are computed as described in Appendix A.

A summary of the numerical procedure to compute $E[W]$ follows.

1. Check that the stability condition (2.26) is satisfied. If it is not satisfied stop. If it is satisfied continue.
2. Use a search procedure to find the $K$ positive roots of $d(\theta)$.
3. Solve the system of equations (2.28)-(2.29) to find $\{p_n(0); n = 0, \ldots, K\}$.
4. Evaluate (2.30) to find $E[W]$.

*d. An Approximation for E[W]*

In this subsection a simple approximation to the model of this section is described.

The limiting distribution of the M/M/K/K queue is of the form

$$\pi(n) = \pi(0)\left(\frac{\lambda}{\upsilon}\right)^n \qquad n = 0, 1, \ldots, K.$$

We approximate the arrival process to the CCS from the M/M/K/K queue by a Poisson process with having rate

$$\upsilon_a = \upsilon \sum_{n=0}^{K} n\pi(n). \tag{2.31}$$

We further approximate the arrival process of outside customers to the CCS by a Poisson process having rate

$$\lambda_a = \lambda[1 - \pi(K)]. \tag{2.32}$$

The total arrival process of customers to the CCS is approximated by a Poisson process having rate $\lambda_t = \upsilon_a + \lambda_a$.

The approximate model of the CCS is an M/G/1 queue with arrival rate $\lambda_t$ and with the service time distribution being the following mixture of the distributions of $X$ and $Y$:

$$P\{S \le t\} = \frac{\lambda_a}{\lambda_t} P\{X \le t\} + \frac{\upsilon_a}{\lambda_t} P\{Y \le t\}. \tag{2.33}$$

The approximation to $E[W]$ is found using the Pollaczek-Khintchine formula

$$W_a = \frac{\lambda_t E\left[S^2\right]}{2(1 - \lambda_t E[S])}. \tag{2.34}$$

The approximate probability that the CCS is idle is

$$P_a(\text{idle}) = 1 - \lambda_t E[S] = 1 - \lambda_a E[X] - \upsilon_a E[Y]. \tag{2.35}$$

### *e. A Comparison of Numerical Results*

A simulation was constructed for the following model of one server in the common channel system. The common channel signaling system is again modeled as a single server with finite waiting room and first-come-first-served service discipline. There is an M/M/K/K queue to model the calls in progress. There is a maximum number of calls allowed in the entire system. Outside arrivals occur according to a Poisson process with rate $\lambda$. An outside arrival is lost if all $K$ servers are busy or there is the maximum number of calls in the entire system. An outside arrival that is not blocked (possibly) queues for an $X$-service by the CCS. *After* its $X$-service is completed it moves to the M/M/K/K queue for a $C$-service. If all $K$ services are busy the customer (call) is lost. After completion of a $C$-service, the customer queues at the CCS for an $Y$-service. When a customer's $Y$-service is completed, the customer leaves the system.

The simulation generates random numbers using LLRANDOM II [cf. Lewis et al. [1981]]. The average work in queue at the time of arrival of a nonblocked outside customer is computed. The fraction of time the CCS is idle is also computed.

Table I presents values for $E[W]$ obtained from the simulation, the model expression (2.30), and the approximate model expression (2.34). All the service times have exponential distributions. The outside arrival rate $\lambda = 1$. The $C$-service times have mean $1/\upsilon$ with $\upsilon = 0.7$. Further, for all cases $E[X] = E[Y]$. For the simulation, the maximum number of customers in the entire system is 1000. The simulation ran for 5000 outside customer arrivals (some of which were blocked). The probability of the CCS being idle is also recorded in Table I for the simulation, the model, and the model approximation.

TABLE I

| | | Average Amount of Work at CCS | | | *P*(Idle CCS) | | |
|---|---|---|---|---|---|---|---|
| $K$ | $E[X]=E[Y]$ | Model | Approximation | Simulation | Model | Approximation | Simulation |
| 2 | 1/2 | 1.22 | 1.19 | 1.31 | 0.30 | 0.30 | 0.29 |
| 3 | 1/2 | 4.01 | 3.55 | 2.38 | 0.12 | 0.12 | 0.15 |
| 4 | 1/2 | 14.67 | 11.33 | 9.13 | 0.04 | 0.04 | 0.04 |
| 2 | 1/3 | 0.30 | 0.29 | 0.40 | 0.53 | 0.53 | 0.53 |
| 3 | 1/3 | 0.50 | 0.47 | 0.51 | 0.42 | 0.42 | 0.43 |
| 4 | 1/3 | 0.68 | 0.59 | 0.63 | 0.36 | 0.36 | 0.35 |
| 5 | 1/3 | 0.80 | 0.64 | 0.67 | 0.34 | 0.34 | 0.32 |

All three models give about the same probability of CCS idleness. The values of $E[W]$ are also very similar for all three models. Further numerical experimentation is needed to study the differences among the three models.

## 3. QUEUEING NETWORK MODELS WITH PRODUCT FORM LIMITING DISTRIBUTION

In this section we describe two simple queueing network models for one server in the signaling network in the spirit of Baskett, et al. [1975]. Queueing network models, while appealing, are often difficult to analyze unless their limiting distribution is of product form. We describe two such models below.

There are two customer types: type 1 is a call setup service and type 2 is a call tear-down service. Outside customers arrive as type 1 customers to the single server CCS.

When a type 1 customer completes service at the CCS, he instantaneously goes to service center 2. Service center 2 is an infinite server queue and represents the calls in progress on the voice/data network; note that we impose no finite limit, $K$, as was done in the earlier model. The service times at service center 2 are independently and identically distributed. When a customer completes service at service center 2, he becomes a type 2 customer and returns to the CCS. After the customer completes his type 2 service, the customer leaves the system.

In the remainder of this section, we describe two queueing network models whose limiting distributions are of product form. The reader may consult Baskett et al. [1975] to obtain ideas for model generalizations and gain some insight into the nature of the modeling restrictions imposed by requiring a product form limiting distribution.

*a. Open Network—FCFS Service at the CCS*

In this model outside customers arrive according to a Poisson process with rate $\lambda$. Type $i$ services at the CCS are independently and identically distributed exponential with mean $1/\mu$; that is, both call setup and call tear-down distributions must have the same mean in this model, which is a restriction. Service times at the infinite-server queue are independent identically distributed exponential with mean $1/\upsilon$.

Let $X_t(i)$ be the number of customers of type $i$ waiting or being served at the CCS at time $t$ and let $N_t$ be the number of customers at Service Center 2 (SC2). It is well known, cf. Baskett et al. [1975], that if $2\lambda < \mu$

$$\pi(n_1, n_2, m) = \lim_{t \to \infty} P\{X_t(1) = n_1, X_t(2) = n_2, N_t = m\}$$

$$= \left[1 - \frac{2\lambda}{\mu}\right]\binom{n_1 + n_2}{n_1}\left(\frac{\lambda}{\mu}\right)^{n_1 + n_2} e^{-\lambda/\upsilon} \frac{\left(\frac{\lambda}{\upsilon}\right)^m}{m!} \tag{3.1}$$

for $n_1, n_2, m$ nonnegative integers.

If the mean service times for type 1 and type 2 customers are not the same, then the limiting distribution will not necessarily be of a product form.

In this model, standard calculations yield that the long run average wait in queue of an arriving type 1 customer to the CCS is $(1/\mu) \lim_{t \to \infty} E[X_t(1) + X_t(2)] = (1/\mu)((2\lambda/\mu)/(1 - 2\lambda/\mu))$ which is the same as the long run average waiting time in queue for an arriving type 2 customer.

To allow for different service time distributions for type 1 and type 2 customers and still have a limiting distribution which is of product form, the service discipline at the CCS must be something like processor-sharing; [cf. Baskett et al. [1975]] which we describe in the next subsection.

*b. An Open Queueing Network with Processor Sharing Service*

The assumptions for this model are the same as before. However type $i$ services are independent identically distributed exponential with mean $1/\mu_i$. A customer's type 1 and type 2 services are independent of each other. Further, the service discipline at the CCS is processor-sharing. Using the same notation as above, if $\rho = \lambda\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right] < 1$, then

$$\pi(n_1, n_2, m) = C\binom{n_1+n_2}{n_1}\left(\frac{\lambda}{\mu_1}\right)^{n_1}\left(\frac{\lambda}{\mu_2}\right)^{n_2}\left(\frac{\lambda}{v}\right)^{m}\frac{1}{m!}e^{-(\lambda/v)} \tag{3.2}$$

where $C = [1-\rho]$ and $n_1, n_2$ and $m$ are nonnegative integers; cf. Baskett et al. [1975].

In this model standard calculations show that the long-run average wait in the CCS queue of an arriving type 1 customer is

$$\lim_{t\to\infty}\left\{\frac{1}{\mu_1}E[X_t(1)] + \frac{1}{\mu_2}E[X_t(2)]\right\} = [1-\rho]^{-1}\left(\frac{\lambda}{\mu_1^2} + \frac{\lambda}{\mu_2^2}\right) \equiv W_Q^1. \tag{3.3}$$

Further the total time spend waiting in queue in the entire system for an arriving type 1 customer is

$$\frac{\rho}{1-\rho}\left[\frac{1}{\mu_1} + \frac{1}{\mu_2}\right] = W_Q^T. \tag{3.4}$$

Hence, the waiting time in queue at the CCS for an arriving customer of type 2 is

$$W_Q^2 = W_Q^T - W_Q^1 = [1 - \rho]^{-1} \frac{2\lambda}{\mu_1 \mu_2}. \tag{3.5}$$

Closed queueing network models similar to models *a* and *b* above may be described in a similar manner.

## 4. A MODEL FOR A COMMON CHANNEL SIGNALLING NETWORK

In this section we describe an example to illustrate an initial model for a common channel signalling network (CCSN). This network is in service to a circuit switching network. The example forms the groundwork for a full-scale model associated with a real network.

The CCSN has several characteristics which will be reflected in its model.

1) Demand for the CCSN is generated by the voice network being served. In our example we will assume that the voice network has already been modeled, and quantities such as the trunk blocking probabilities and arrival rates of calls requiring each trunk can be obtained.

2) The network is large and heavily used by many different sources. As a result the dependence between servers is a secondary effect; (cf. Kelly [1986]) Thus, the signaling load generated by each trunk s modeled as independent from trunk to trunk.

### a. *An Example of a Signaling Network Model*

For purposes of illustration we will consider a network containing three nodes: labeled 1, 2, 3 and connected as shown below.

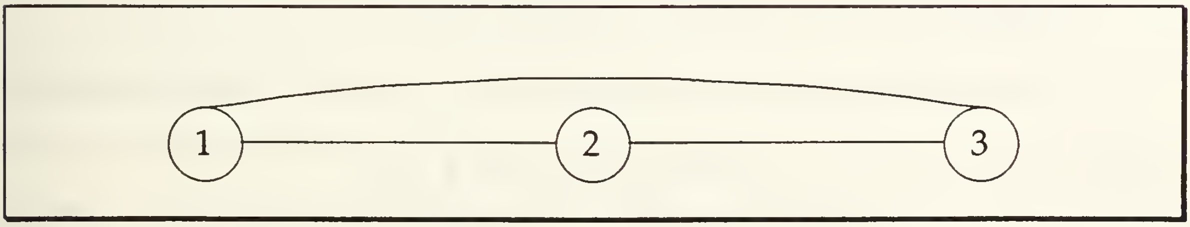


**Figure 1**

We will consider calls for three source-destination pairs.

| Source | Destination | External Call Arrival Rate for Source Destination Pair |
|---|---|---|
| 1 | 2 | $\alpha_{12}$ |
| 2 | 3 | $\alpha_{23}$ |
| 1 | 3 | $\alpha_{13}$ |

The routing for calls between source destination pairs is as follows: The calls for source-destination $(s,d)$ pair (1,2) can only use the trunk connecting 1 and 2, denoted [1,2]. The calls for $(s,d)$ pair (2,3) can only use the trunk connecting 2 and 3, i.e. [2,3]. The calls for $(s,d)$ pair (1,3) have a preferred route using the trunk connecting 1 and 3, [1,3]. If the trunk [1,3] is blocked, the call will attempt to use an alternate route which consists of [1,2] and [2,3].

We will make the following assumptions concerning the CCSN.

(a) There is a signaling link for each trunk. The time required for a signaling operation is a random variable independent of the circuit switching congestion. In principle, this may be questionable, and is subject to change.

(b) The common channel signaling network for each node can be modeled as independent single servers.

(c) Each attempt for a call setup on a trunk between $i$ and $j$ (whether successful or not) generates a call setup service for the signaling servers at nodes $i$ and $j$.

(d) When a call between a source-destination pair is completed, a call tear-down service is required by the signaling server at each node along its route.

The signaling server at each node will be modeled using the M/G/1 queueing model of (2.31)–(2.34). This model will require the effective arrival rate of call setup attempts (whether or not successful) and the arrival rate of carried calls. It also requires a service time characterization; this is presumably the time for a packet to pass from the originating node to the next signaling node.

Let $B_{ij}$ be the probability that voice trunk $[i,j]$ is blocked. The arrival rates of call setup attempts is approximately computed as follows; cf. Kelly [1989].

| Trunk | Effective Arrival Rate of Call Setup Attempts |
|---|---|
| [1,2] | $e_{12} = \alpha_{12} + \alpha_{13} B_{13}$ |
| [2,3] | $e_{23} = \alpha_{23} + \alpha_{13} B_{13}[1-B_{12}]$ |
| [1,3] | $e_{13} = \alpha_{13}$ |

To explain $e_{12}$, note that the effective arrival rate of call setups on trunk [1,2] is the direct call rate $\alpha_{12}$, plus the calls that attempt to go from 1 to 3 but are blocked, and hence rerouted on [1,2]. Likewise, $e_{23}$ is the sum of the rate of outside calls attempting to access 3 from 2, plus the calls originating from 1 and intended for 3 that are blocked on [1,3] but not blocked on [1,2]. All other effective arrival rates are 0. The arrival rate of carried calls to trunk $[i,j]$ is computed as follows

| Trunk | Arrival Rate of Carried Calls |
|---|---|
| [1,2] | $c_{12} = e_{12}\,[1-B_{12}]$ |
| [2,3] | $c_{23} = e_{23}\,[1-B_{23}]$ |
| [1,3] | $c_{13} = e_{13}\,[1-B_{13}]$ |

All other effective arrival rates are 0.

The model for work at the signaling server at node $i$ is an M/G/1 queue with arrival rate

$$\xi_i = \sum_j \left(e_{ij} + c_{ij}\right) + \sum_k \left(e_{ki} + c_{ki}\right) \tag{4.1}$$

where the summations are over all nodes $j$ and $k$. The service time distribution is a mixture of distributions

$$P\{S_i \le t\} = \frac{\sum_j e_{ij} + \sum_k e_{ki}}{\xi_i} \cdot P\{X \le t\} + \frac{\sum_j c_{ij} + \sum_k c_{ki}}{\xi_i} \cdot P\{Y \le t\} \tag{4.2}$$

where $X$ is the length of a call setup service and $Y$ is the length of a call tear-down service. If $\overline{W}_i$ is the average delay at the signaling server at node $i$, then, by familiar M/G/1 formulas, see Kleinrock [1975],

$$\overline{W}_i = \frac{\xi_i E[S_i^2]}{2(1 - \xi_i E[S_i])} \quad \text{if } \xi_i E[S_i] \le 1. \tag{4.3}$$

The expected delay on the signaling network for attempts to set up a call between a source-destination pair can now be computed. In what follows we will assume that a call setup attempt using trunk $[i,j]$ requires a service at the signaling server both nodes $i$ and $j$ with possible queueing at each node; the $X$-service at node $i$ must be completed before the service at node $j$ begins. This assumption can be modified for specific signaling network protocols. For example, a call from source 1 to destination 3 has an initial call setup service at nodes 1 and 3; if trunk [1,3] is blocked another call setup service is required at nodes 1 and 2; if trunk [1,2] is not blocked, then an additional call setup service is required at nodes 2 and 3. Hence the expected delay on the signaling network for an attempt to set up a call from source 1 to destination 3 is

$$\begin{aligned} E[D_{13}] &= \overline{W}_1 + E[X] + \overline{W}_3 + E[X] + B_{13}\left[\overline{W}_1 + E[X] + \overline{W}_2 + E[X] + [1 - B_{12}]\left[\overline{W}_2 + \overline{W}_3 + 2E[X]\right]\right] \\ &= \overline{W}_1 + \overline{W}_3 + 2E[X] + B_{13}\left[\overline{W}_1 + \overline{W}_2 + 2E[X] + [1 - B_{12}]\left[\overline{W}_2 + \overline{W}_3 + 2E[X]\right]\right] \qquad (4.4) \end{aligned}$$

The time to tear-down a call between 1 and 3 can also be computed. In what follows we will assume that a call tear-down using trunk $[i,j]$ requires a

$Y$-service at both node $i$ and node $j$. For example a call from source 1 to destination 3 has a call tear-down service at nodes 1 and 3 with the conditional probability that the carried call used link [1,3]; this conditional probability is

$$[1 - B_{13}]\{[1 - B_{13}] + B_{13}[1 - B_{12}][1 - B_{23}]\}^{-1} \equiv p_1. \tag{4.5}$$

It has a tear-down service at nodes 1, 2, and 3 with the conditional probability that the call uses both trunks [1,2] and [1,3]; this conditional probability is

$$\{B_{13}[1 - B_{12}][1 - B_{23}]\}\{[1 - B_{13}] + B_{13}[1 - B_{12}][1 - B_{23}]\}^{-1} \equiv p_2. \tag{4.6}$$

Hence the expected delay tearing down a call on the signaling network is

$$p_1[\overline{W}_1 + \overline{W}_3 + 2E[Y]] + p_2[\overline{W}_1 + \overline{W}_2 + \overline{W}_3 + 3E[Y]] = \overline{W}_1 + \overline{W}_3 + p_2(\overline{W}_2 + E[Y]) + 2E[Y].$$

## 5. CONCLUSION

In this paper we have considered several possible approaches to modeling a single server in a common channel signaling network. The modeling assumptions required by queueing network models with product form solutions appear restrictive. An M/G/1 approximation to a more detailed model of work at a common channel server appears to be adequate for practical purposes. An approach to modeling a network of common channel servers is suggested.

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## APPENDIX A

In this section we will describe the arguments leading to the expressions for $\sum_{n=1}^{K+1} b_n(0)$ and $d(0)$ in Section 3. For simplicity, we will illustrate them for the case $K = 2$.

If $K = 2$, then

$$D(\theta) = \begin{bmatrix} \theta - \lambda & \upsilon\hat{Y}(\theta) & 0 \\ \lambda\hat{X}(\theta) & (\theta - (\lambda + \upsilon)) & 2\upsilon\hat{Y}(\theta) \\ 0 & \lambda\hat{X}(\theta) & (\theta - 2\upsilon) \end{bmatrix}; \tag{A.1}$$

$$B_1(0) = \begin{bmatrix} p_0(0) & \upsilon & 0 \\ p_1(0) & -(\lambda + \upsilon) & 2\upsilon \\ p_2(0) & \lambda & -2\upsilon \end{bmatrix}; \tag{A.2}$$

$$B_2(0) = \begin{bmatrix} -\lambda & p_0(0) & 0 \\ \lambda & p_1(0) & 2\upsilon \\ 0 & p_2(0) & -2\upsilon \end{bmatrix}; \tag{A.3}$$

$$B_3(0) = \begin{bmatrix} -\lambda & \upsilon & p_0(0) \\ \lambda & -(\lambda + \upsilon) & p_1(0) \\ 0 & \lambda & p_2(0) \end{bmatrix}. \tag{A.4}$$

Adding the first row of $B_i(0)$ to the second row and then adding the resulting second row to the third row results in the matrices

$$B_1^0(0) = \begin{bmatrix} p_0(0) & \upsilon & 0 \\ p_0(0) + p_1(0) & -\lambda & 2\upsilon \\ p_0(0) + p_1(0) + p_2(0) & 0 & 0 \end{bmatrix} \tag{A.5}$$

$$B_2^0(0) = \begin{bmatrix} -\lambda & p_0(0) & 0 \\ 0 & p_0(0)+p_1(0) & 2\upsilon \\ 0 & p_0(0)+p_1(0)+p_2(0) & 0 \end{bmatrix} \tag{A.6}$$

$$B_3^0(0) = \begin{bmatrix} -\lambda & \upsilon & p_0(0) \\ 0 & -\lambda & p_0(0)+p_1(0) \\ 0 & 0 & p_0(0)+p_1(0)+p_2(0) \end{bmatrix} \tag{A.7}$$

Since det $B_i(0) = \det B_i^0(0)$ and the minor of the entry $\sum_{i=0}^{2} p_i(0)$ is an upper diagonal matrix, it follows that

$$\sum_{n=1}^{3} b_n(0) = \left[\sum_{i=0}^{2} p_i(0)\right]\left[\lambda^2 + 2\lambda\upsilon + 2\upsilon^2\right]. \tag{A.8}$$

To find an expression for $d(0)$, note that

$$\begin{aligned} d(0) &= \frac{d}{d\theta}\det D(\theta)\Big|_{\theta=0} \\ &= \sum_{i=1}^{3} \det D_i(\theta)\Big|_{\theta=0} \end{aligned} \tag{A.9}$$

where $D_i(\theta)$ is the matrix $D(\theta)$ with the same entries as $D(\theta)$ except that the i[th] row contains the derivatives of the functions of the i[th] row of $D(\theta)$; cf. Apostle [1969].

In the case of $K = 2$

$$D_1(0) = \begin{bmatrix} 1 & -\upsilon E[Y] & 0 \\ \lambda & -(\lambda+\upsilon) & 2\upsilon \\ 0 & \lambda & -2\upsilon \end{bmatrix} \tag{A.10}$$

$$D_2(0) = \begin{bmatrix} -\lambda & \upsilon & 0 \\ -\lambda E[X] & 1 & -2\upsilon E[Y] \\ 0 & \lambda & -2\upsilon \end{bmatrix} \tag{A.11}$$

$$D_3(0) = \begin{bmatrix} -\lambda & \upsilon & 0 \\ \lambda & -(\lambda + \upsilon) & 2\upsilon \\ 0 & -\lambda E[X] & 1 \end{bmatrix} \tag{A.12}$$

$$\begin{aligned} \det D_1(0) &= 1 \det\begin{bmatrix} -(\lambda + \upsilon) & 2\upsilon \\ \lambda & -2\upsilon \end{bmatrix} + \upsilon E[Y] \det\begin{bmatrix} \lambda & 2\upsilon \\ 0 & -2\upsilon \end{bmatrix} \\ &= 1 \det\begin{bmatrix} -\upsilon & 0 \\ \lambda & -2\upsilon \end{bmatrix} + \upsilon E[Y] \det\begin{bmatrix} \lambda & 2\upsilon \\ 0 & -2\upsilon \end{bmatrix} \\ &= 1\left(2\upsilon^2\right) + \upsilon E(Y)[-2\lambda\upsilon] \end{aligned} \tag{A.13}$$

$$\begin{aligned} \det D_2(0) &= \lambda E[X] \det\begin{bmatrix} \upsilon & 0 \\ 1 & -2\upsilon \end{bmatrix} + 1 \det\begin{bmatrix} -\lambda & 0 \\ 0 & -2\upsilon \end{bmatrix} + 2\upsilon E[Y] \det\begin{bmatrix} -\lambda & 0 \\ 0 & \lambda \end{bmatrix} \\ &= -\lambda E[X]\left[2\upsilon^2\right] + \lambda(2\upsilon) - 2\upsilon E[Y]\lambda^2 \end{aligned} \tag{A.14}$$

$$\begin{aligned} \det D_3(0) &= \lambda E[X] \det\begin{bmatrix} -\lambda & 0 \\ \lambda & 2\upsilon \end{bmatrix} + 1 \det\begin{bmatrix} -\lambda & \upsilon \\ \lambda & -(\lambda + \upsilon) \end{bmatrix} \\ &= \lambda E[X] \det\begin{bmatrix} -\lambda & 0 \\ \lambda & 2\upsilon \end{bmatrix} + 1 \det\begin{bmatrix} -\lambda & \upsilon \\ \lambda & -\lambda \end{bmatrix} \\ &= \lambda E[X][-2\lambda\upsilon] + \lambda^2 \end{aligned} \tag{A.15}$$

Thus

$$\begin{aligned} d(0) &= \sum_{i=1}^{3} \det D_i(0) \\ &= \lambda^2 + 2\upsilon\lambda + 2\upsilon^2 - \lambda E[X]\left[2\lambda\upsilon + 2\upsilon^2\right] - \upsilon E[Y]\left[2\lambda\upsilon + 2\lambda^2\right] \end{aligned} \tag{A.16}$$

## APPENDIX B. THE DEFENSE COMMUNICATIONS AGENCY CIRCUIT SWITCHED NETWORK DESIGN/ANALYSIS MODEL (CSNDAM)

The Defense Communications Agency is currently using a computer model called the CSNDAM (Circuit Switched Network Design/Analysis Model) "to design and maintain optimal cost-effective circuit switched networks"; cf. Defense Communications Agency [1989].

The CSNDAM can be used to model traffic on a voice network. This model for a voice network could then be used to generate demand in a model for the common channel signaling network as outlined in Section 4. In this appendix we discuss two issues concerning using the CSNDAM to model the voice network which generates demand on a common channel signaling network.

The first issue is minor. The CSNDAM model output is in terms of load measured in Erlangs. Thus, to obtain arrival rates, the loads must be divided by the expected call holding time. For example, if $\frac{1}{\upsilon}$ is the expected call holding time and $a_{ij}$ is the CSNDAM computed offered load (in Erlangs) on trunk $[i,j]$, then $a_{ij}\upsilon$ is part of the call setup arrival rate for nodes $i$ and $j$.

The second issue is more important. In its computation of offered load, CSNDAM subtracts load on a link due to calls that unsuccessfully attempt to use that link for part of their path from source to destination. However, a signal is generated on the common channel signaling network even if a call set up attempt is unsuccessful. Hence, the arrival rate of call set up signals for node $i$ not only involves the offered load for all traffic passing through node $i$

but also the load through node *i* generated by calls that unsuccessfully attempt to use a path through node *i*. Thus, it appears that in order to use the CSNDAM model to generate demand for the common channel signaling network, the CSNDAM model would have to be modified to calculate the load on the signaling network due to lost calls.

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